Nonlinear Mixed-Effects Model for the Evaluation and Prediction of Pavement Deterioration

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Abstract: Pavement deterioration models are important inputs for pavement management systems (PMS). These models are based on the study of performance data, and they provide the evolution law of pavement deterioration. Performance data consist of observations of pavement section conditions, and are collected through several follow-up campaigns on road networks. To characterize the pavement deterioration process, several statistical methods have been developed at the Laboratoire Central des Ponts et Chaussées (LCPC). However, these methods are suboptimal for modeling the evolution of pavement deterioration, as they ignore unit-specific random effects and potential correlation among repeated measurements. This paper presents a nonlinear mixed-effects model enabling accounting for the correlation between observations on the same pavement section. On the basis of this nonlinear mixed-effects modeling, we investigate and identify structural and climatic factors that explain differences in the parameters between pavement sections, and quantify the impact of these factors on pavement evolution. The proposed model provides a good fit for describing the evolution law of different pavement sections. The performance of this model is assessed using simulated and real data. **DOI: 10.1061/(ASCE)TE.1943-5436.0000257.** *© 2012 American Society of Civil Engineers.*

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Introduction

The optimization of road maintenance management tasks, over both the medium and long terms, requires knowledge of the set of factors that could influence the pavement condition. Under the repeated application of traffic loads, such factors are divided into two categories: the pavement structure (thickness of the various layers) and the climatic conditions (height of precipitation) [Laboratoire Central des Ponts et Chaussées-Service d'études sur les transports, les routes et leurs aménagements (LCPC-SETRA) 1994]. To identify these factors, statistical methods for analyzing experimental data from repeated measurements have been introduced (Lorino et al. 2006; Lepert et al. 2004, 2003; Leroux et al. 2004). They enable describing the evolution curve of cracking on particular pavement sections according to the pavement section age and with respect to one or several explanatory variables.

These modeling methods are classified into two categories: nonlinear regression methods, called direct, indirect methods (Leroux 2003; Lepert and Riouall 2002) and statistical methods resulting from application of survival laws theory (Cox proportional hazards and parametric models) (Courilleau and Marion 1999; Rèche 2004). The advantage of using mathematical models is that they facilitate the analysis and interpretation of the observed data because they describe the evolution law as a function of only a few parameters that can be statistically compared.

However, these previous analyses cannot exactly predict the measured data because of unaccounted correlation between observations on the same pavement section. Also, they cannot test whether the measurement error is significant (Khraibani et al. 2009). To this end, we introduce the nonlinear mixed-effects model to predict future pavement conditions.

For repeated measurements data, mixed-effects models offer a flexible framework in which population characteristics are modeled as fixed effects and unit-specific variation is modeled as random effects. Linear mixed-effects (LME) models (Laird and Ware 1982; Ware 1985; Diggle et al. 1994) and nonlinear mixed-effects (NLME) models (Davidian and Giltinan 1995; Vonesh and Chinchilli 1997) are widely used in longitudinal data analysis.

Recent literature on reliability contains many papers that applied mixed-effects approaches to model a wide variety of degradation data. Archilla and Madanat (2001) and Onar et al. (2006) proposed a linear mixed-effect model for pavement application. Yuan and Pandey (2009) used a nonlinear mixed-effects model for monitoring and predicting degradation in nuclear piping systems.

The overall objectives of this paper are to (1) develop a nonlinear mixed-effects model for describing pavement section behavior as a function of time, taking individualization into account; (2) employ a logistic function to model the sigmoid evolution law of pavement cracking; and (3) examine the effects of the pavement structure and the climatic conditions factors on pavement behavior by incorporating covariates into the model. Finally, the analysis

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was performed with the statistical software S-PLUS and its library for nonlinear mixed-effects models (NLME) (Pinheiro and Bates 2000).

Methodology

Data Sets

The NLME model described in this paper was applied to two types of pavement cracking database. The first type corresponded to slowly degraded test section data that comes from a cooperation between the Laboratoire Central des Ponts et Chaussées (LCPC) and the Ministère des Transports du Québec (MTQ). Noting that, the aim of the cooperation project was to study the pavement behavior in the winter. The second database corresponded to simulated data, which allowed us to investigate the influence of structural factors and mechanical properties on the evolution law of pavement deterioration.

MTQ Data

Pavement test sections were constructed in Québec around 1998 and their cracking behavior was monitored over nine years. The 45 pavement sections of the test set are shown in Fig. 1. Notice that each section was visually inspected between 2 and 12 times per year. On-site operators established a cartographic representation of the cracking, which then served as a basis for the precise laboratory measurement of a visible crack length. The total measured crack length was expressed in millimeters per square meter. This approach enabled not only a comparison of test sections that may contain different lane widths and different number of lanes, but also observation of crack progression better than a simple calculation of the number of cracks.

However, the data set was represented by longitudinal, irregularly spaced, and unbalanced plots. For all sections in this data, thermal cracking appeared after pavement maintenance but with a very low cracking rate as a phenomenon of premature cracking. After a period of five years, cracks progressed at a higher rate; within a few months, or even weeks, the majority of the cracks formed. As known, pavements exposed to harsh winter climates experience transverse thermal cracking earlier in their life cycle. Consequently, the two covariates associated with the response variable (percent of cracking) are the time of observation and the averaged annual height of precipitation (H_p).

The measurements of the same section are connected with lines. For each section, a distinct nonlinear increase of pavement cracking with age is shown, but a considerable variation exists among sections.

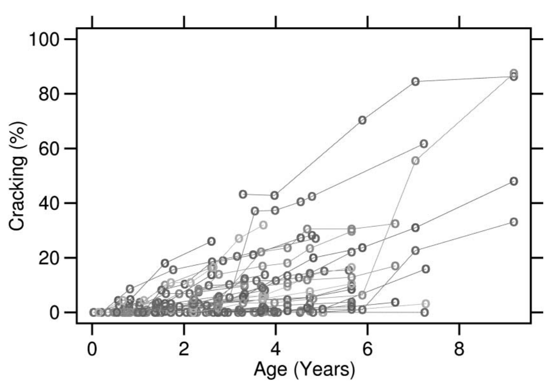

Fig. 1. Observed thermal cracking (%) versus age (years) for 45 pavement sections

Simulated Data

The data set was generated using a fatigue-crack propagation model whose random parameters followed a Weibull probability distribution. These simulated data contained 100 maintained sections. For each section, 90 to 120 cracking measurements varying from 0% to 100% were considered. These cracking measurements depended on both the initial pavement layer and the surface layer (overlayer) and all fatigue cracking measurements for each pavement section were assumed to return to 0% after maintenance. In other words, the nonlinear behavior of the initial pavement layer was taken into account by subdividing the layer into two successive layers (bottom and interface layer) having mechanical characteristics that vary as a function of the stress and the thickness of the initial layer. Similarly, when overlaying the initial layer, the nonlinear behavior of the maintained pavement was taken into account by subdividing the overlayer into two successive layers (interface and surface layer) with mechanical characteristics that vary as a function of the stress and thickness of the initial and surface layers. Hence, considering a fixed elastic modulus at 16,000 MPa, this indicator was based on three uncorrelated covariates

- h in [0.335; 0.465]: thickness of the initial layer expressed in m,
- c_t in [65; 130]: time of maintenance expressed in months, and
- h_e in [0.085; 0.13]: thickness of the surface layer expressed in m.

Generalized Nonlinear Mixed-Effects Models

As noted in the introduction, the nonlinear mixed-effects framework is widely used in describing a nonlinear relationship between a response variable and parameters and covariates in the repeated measurements data that are grouped by a cluster factor. NLME models were initially proposed, in biostatistics literature, by Lindstrom and Bates (1990), Pinheiro and Bates (1995), and Davidian and Giltinan (1995). This study follows the generalized nonlinear mixed-effects models proposed by Lindstrom and Bates (1990).

For pavement section i with n_i repeated measurements, the generalized NLME model for pavement cracking data can be expressed as

$$y_{ij} = f(\beta_i; a_{ij}, x_{ij}) + e_{ij}, \qquad i = 1, \ldots., j, \ldots m; \qquad j = 1, \ldots, n_i \tag{1}$$

where y_{ij} = measured value of the deterioration for section i at time j; a_{ij} = age (years) for the ith section on time j, f = nonlinear function relating the response variable to age and to other possible covariates x_{ij} varying with individual and time, β_i = vector with the parameters of nonlinear function, and e_{ij} = normally distributed within-section error term. The parameter β_i varies from one section to another to account for intra and intersection variation and is modeled as

$$\beta_i = A_i\beta + B_i b_i + \varepsilon_i, \qquad b_i \sim N(0, \sigma_i^2), \qquad \varepsilon_i \sim N(0, \sigma^2\Lambda_i) \tag{2}$$

where

- $\beta = p \times 1$ vector of fixed effects parameters, common to all sections,
- $b_i = q \times 1$ vector of random effects associated with the ith section,
- ε_i = within group errors vector assumed to be independently distributed with zero mean and variance-covariance matrix $\sigma^2\Lambda_i$, where Λ_i is the identity matrix,
- p = number of fixed parameters in the model,
- q = number of random parameters in the model, and
- $(\sigma_i)^2$ = variance-covariance matrix for the random effects.

A_i and B_i were design matrices for the fixed and random effects, respectively, where A_i was adequate to describe the possible influence of controlled factors in the parameters β of the model and B_i described the variation between observations on each specific section.

Estimated parameters and modeling variance-covariance structure were developed by using the maximum likelihood (ML) method (Pinheiro and Bates 1995). Criteria and test procedures were used for comparing alternative models.

In general, likelihood ratio tests are useful for choosing between two models, where one model is a subset of the other. Akaike's information criterion (AIC) (Akaike 1974) and Bayesian information criterion (BIC) (Schwarz 1976) were used to compare several alternative models. Formulas for their computation are as follows. $\mathrm{AIC} = -2\ln L(\Theta) + 2k$, where $L(\Theta)$ = maximized likelihood function, k = number of model parameters. $\mathrm{BIC} = -2\ln L(\Theta) + k\ln(n)$, where n = number of observations, equivalently the sample size; k = number of free parameters to be estimated, and $L(\Theta)$ = maximized value of the likelihood function for the estimated model. The smallest value for both criteria indicates the best fit.

Working Procedure to Fit NLME Model

To describe the evolution law of pavement behavior using NLME model, the analysis consisted of three main steps.

- To avoid convergence problems attributable to over parameterization, we will temporarily ignore the random-effect was temporarily ignored and a single nonlinear fixed-effect model was fit using nonlinear least square method. If no convergence was obtained, other starting values were attempted.
- Next, the random effects parameters, as well as possible combinations, were introduced. The deviations of the individual pavement sections from the fixed-effect parameters were represented in a random-effect plot. Diagnostics plots were used to evaluate the model fit; for example, the normality of the random effects were tested and the homogeneous spread around the zero line of the residuals was checked.
- Finally, the third step was summarized by incorporating of several factors into the NLME model and testing and analyzing the effect of these factors on pavement behavior.

Results

MTQ Data

The model described above was applied to the MTQ database with the objective of modeling the behavior of flexible pavements. The MTQ database was chosen not only for its availability, but also for its representativeness of thick bituminous pavement, largely used in France and for its behavior in a severe climate, which maximizes the distress. The form adopted for predicting the cracking measurements was assumed as a sigmoïdal. Therefore, pavement sections were described by logistic models

$$y_{ij} = \frac{A}{1 + \exp[-(\frac{(t_{ij} - B)}{C})]} + e_{ij}(3) \tag{3}$$

where y_{ij} = percentage of cracking for the ith section at the jth measurement time t_{ij} and $i = 1, \ldots, 45$, $j = 1, \ldots, n_i$. e_{ij} = within-section error term associated with the j^h measurement on the ith section and is assumed to have a Gaussian distribution with zero mean and variance σ^2. The parameter A corresponded to the value of the horizontal asymptote (the limit of cracking growth) at which roads are

Table 1. Statistics of Nonlinear Fixed-Effect Model

	Value	STD error	DF	t-value	P-value
B	7.648	0.433	216	17.643	< 0.0001
C	1.472	0.140	216	10.466	< 0.0001

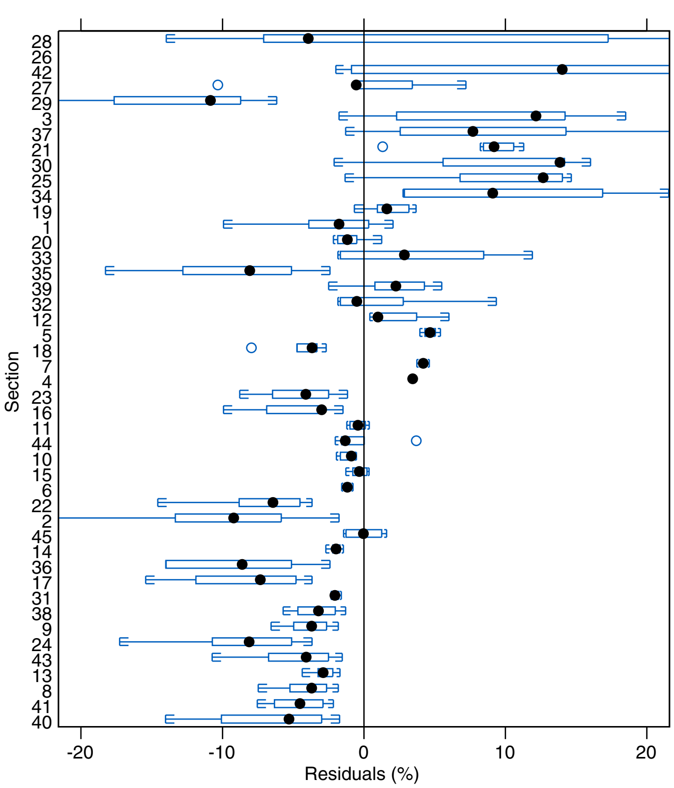

Fig. 2. Box plot of residuals from the nonlinear fixed-effect model

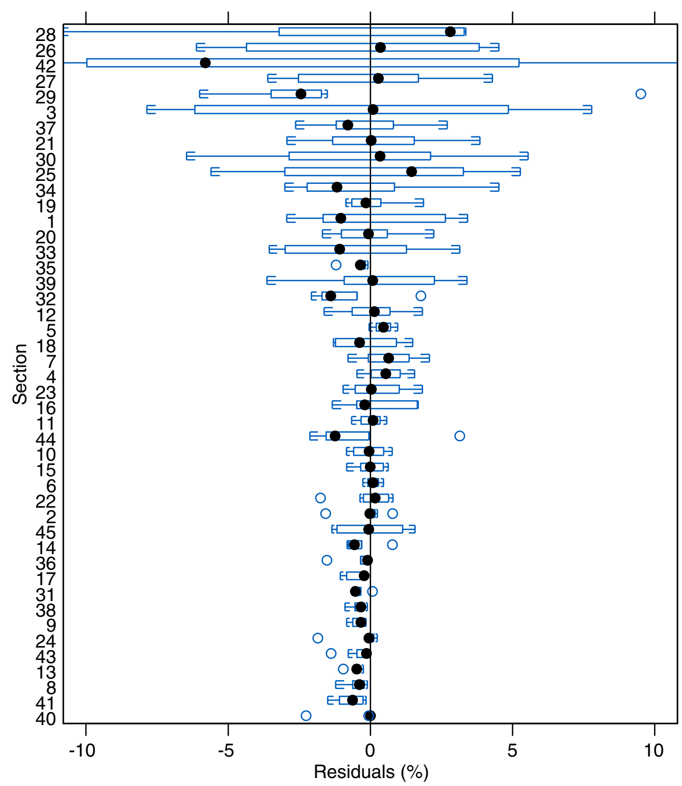

Fig. 3. Box plot of residuals from the nonlinear mixed-effect model

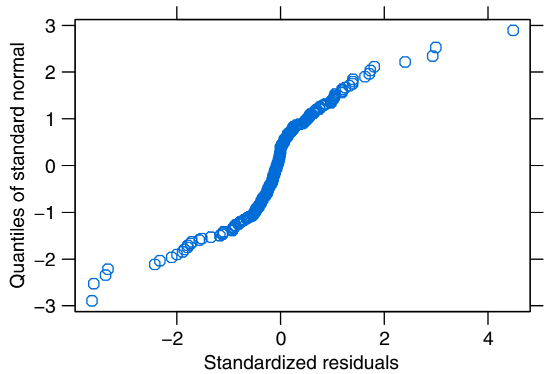

Fig. 4. Normal QQ plot of residuals

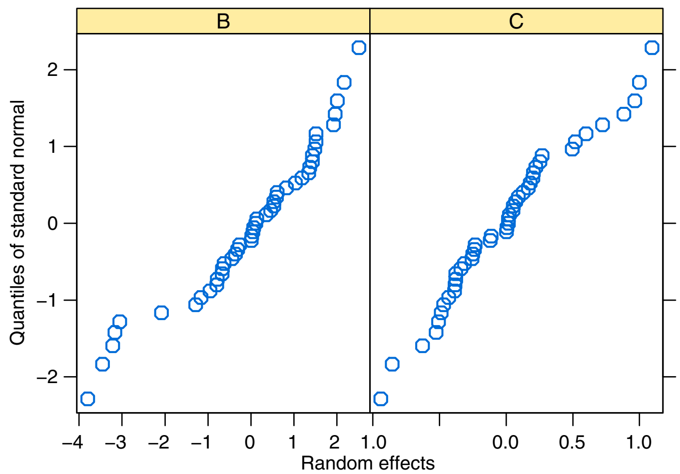

Fig. 5. Normal QQ plot of random effects

completely degraded, and was set to 100% of cracking. The parameter B is the midpoint, the time at which $y_{ij} = A/2 = 50\%$. C is the scale parameter representing the distance on the time axis between the midpoint and the point at which the response is $A/(1+e^{-1}) = 73\%$ of cracking.

The first step was to determine appropriate starting values for estimating model parameters. A suitable starting estimate for B was the average time at which 50% of cracking was reached, and a reasonable starting value for C was the difference between the average times at which 50% and 73% of cracking were reached, respectively.

We began with fitting a fixed nonlinear model (3) to the entire data set. The least square estimates of the parameters were $B = 8.536$, $C = 1.935$. The P-values indicated that the B and C parameters were significant (< 0.0001) and both batches were statistically different; therefore, the model was correctly parameterized. The residual standard error (S.E.) was 10.630. Fig. 2 shows the boxplot of residuals from model (3), and that the residuals tended to be negative for some sections and positive for others, and the plots had different variations according to the residual S.E.

To account for variations on the same pavement section, random components were introduced into model (3), yielding the following nonlinear mixed-effects model

$$y_{ij} = \frac{100}{1 + \exp[-(\frac{t_{ij}-(B+b_i)}{(C+c_i)})]} + e_{ij} \tag{4}$$

with the assumptions $(b_i, c_i) \sim N(0, (\sigma_i)^2)$, $e_{ij} \sim N(0, \sigma^2)$ and $b_1, \ldots, b_n$, $c_1, \ldots, c_n$, $e_1, \ldots, e_n$, independent (with the number of section $n = 45$). B was replaced by $B + b_i$ to account for the correlation between observations on the intra-individual variability in the midpoint time. B was called the fixed-effect and b_i was called the random-effect and represented the individual section departure from the average time of the midpoint. Similarly, the fixed-effect C represented the mean level of the growth time for the population

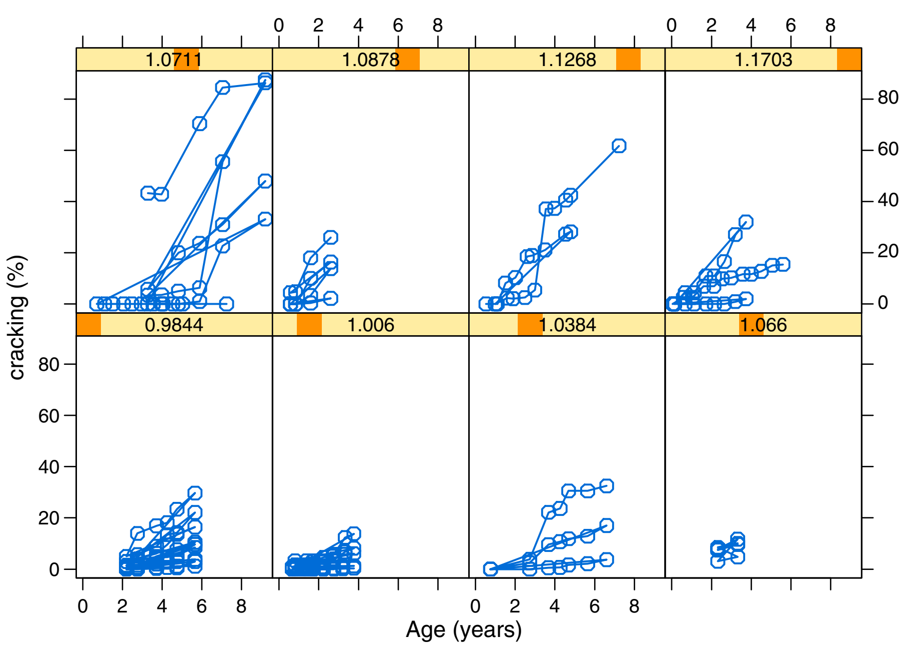

Fig. 6. Observed fatigue cracking (%) versus age (years) for the 45 pavement sections by climatic conditions [average annual height of precipitation (H_p) indicated at the top of each graphic cell]

and c_i was the individual section departure from the mean level of the growth time.

The data were fitted with model (4), assuming that both random effects were added to the formula. The parameter values, estimated by maximum likelihood (ML), are given in Table 1. The P-values of B and C remained very small (< 0.0001) and indicated that both parameters were significant at the 5% level.

A rather weak correlation (0.354) was found between the midpoint (B) and the shape parameter (C) indicating that the two random effects from model (4) are required. Significant section-to-section variation in B and C may reflect that the velocity of propagation differs from one section to another, in which each section has its own strength at a given moment.

In comparison with the estimates of model (3), the estimates of the residual S.E. in model (4) decreased drastically from 10.630 to 3.241.

Fig. 3, showing the box plot of the residuals in model (4), indicates that the residuals are approximately centered at zero, and with several outlying observations. whereas the residuals of model (3) have alternating sign and are much larger. Moreover, the normality of the measurement errors was checked using the normal QQ plot of the residuals (Fig. 4). Fig. 4 indicates no significant deviation from the normality assumption. In addition, Fig. 5 shows the normality of the random effects. The assumption of normality seems reasonable for both random effects.

Table 2. Statistics of Nonlinear Mixed-Effect Model with Covariate (H_p)

	Value	STD error	DF	t-value	P-value
B	22.863	8.056	216	2.838	0.005
β	−14.480	7.569	216	17.643	0.047
C	2.258	2.643	216	10.466	0.394
χ	−0.725	2.493	216	15.321	0.771

Table 3. Statistics of Nonlinear Mixed-Effect Model with Covariate (H_p) [Model 5]

	Value	STD error	DF	t-value	P-value
B	22.863	8.056	216	2.838	0.005
β	−14.480	7.569	216	17.643	0.047
C	2.258	2.643	216	10.466	0.394

Having established the most appropriate mixed-effects model for the cracking measurements, the next step was to incorporate the relevant covariate height of precipitation (H_p) into model (4). The H_p variable is an important covariate for explaining section-to-section variation (interindividual variability), as Fig. 6 seems to indicate. Consequently, all parameters could be influenced by this variability. Table 2 shows the results from including H_p in the model as a covariate to explain the systematic among-section variability in B and C parameters. They indicated that the estimated parameters B and β (β is the parameter associated with H_p in B) were significant at the 5% level (P-value < 0.05), whereas the parameters C and χ (χ is the parameter associated with H_p in C) were not significant (P-value > 0.05) and therefore do not explain the variation of pavement cracking depending on H_p.

Next, the backward elimination approach was used to select the final model that fits the MTQ data with experimental conditions (climatic factor). Then, non-significant terms were removed from the model recursively (χ in this case).

The final model for the prediction of cracking measurements is

$$y_{ij} = \frac{100}{1 + \exp[-(\frac{t_{ij}-(B+\beta\times H_p+b_i)}{(C+c_i)})]} + e_{ij}(5) \qquad (5)$$

where β = height of precipitation effect on the mean growth time B. The significance of this term was assessed using the Wald tests (see Table 3).

Because β was negative, the midpoint of the specific section decreased with increasing values of H_p and, consequently, the pavement sections deteriorated faster. Fittings model (5) were very close to observations (see Fig. 7), reflecting the intra-individual variability in the B and C parameters and the interindividual variability in the parameter B once H_p were taken into account.

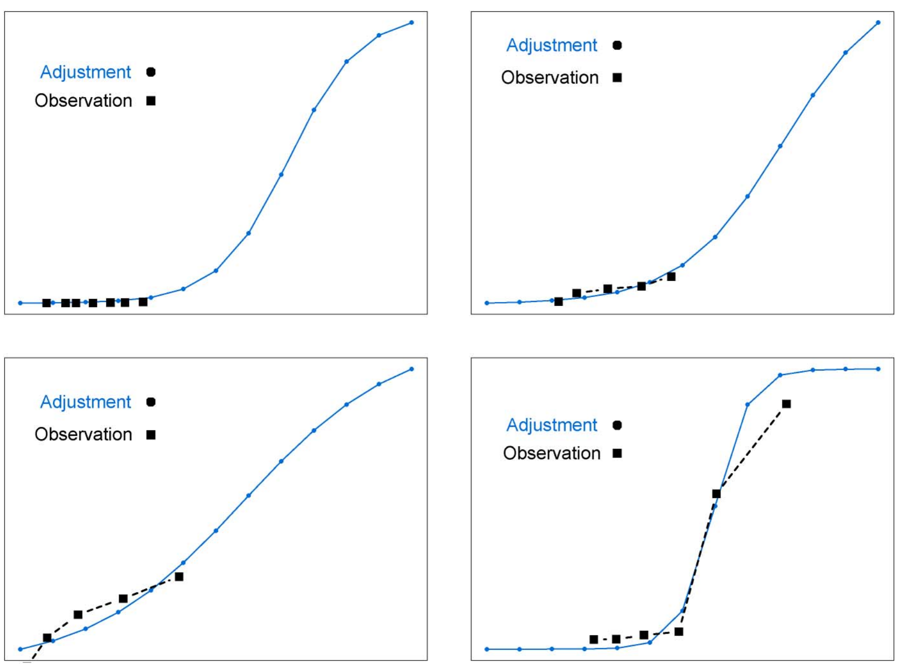

Fig. 7. Cracking measurement is plotted against age (years) from the MTQ data, along with the individual fitted curves for each of four sections from the 45 pavement sections

Table 4. Likelihood Ratio Tests (LRT) for Nonlinear Mixed-Effects Models

Model	Random parameters	AIC	BIC	Log-likelihood	Test	LRT	P-value
1	B, C	30777	30820	−15,382		—	
2	B	51823	51852	−25,909	1 versus 2	21050	< 0.0001
3	C	84752	84781	−42,372	1 versus 3	53979	< 0.0001
4	None	86041	86063	−43,017	1 versus 4	55270	< 0.0001

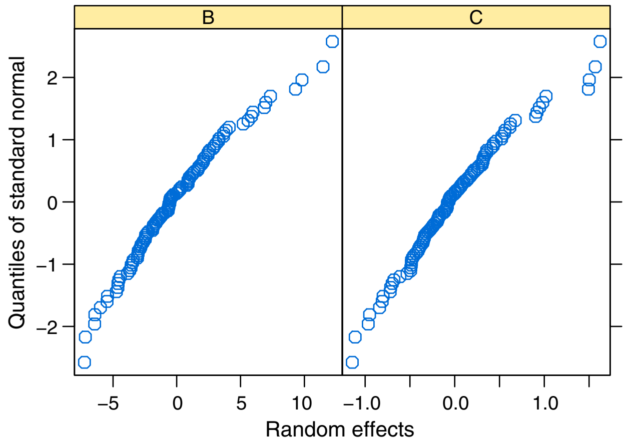

Fig. 8. Normal QQ plot of random effects

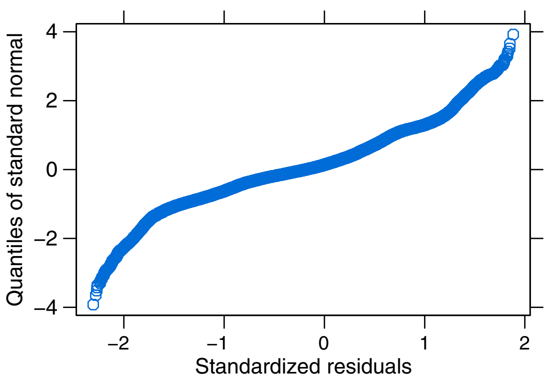

Fig. 9. Normal QQ plot of residuals

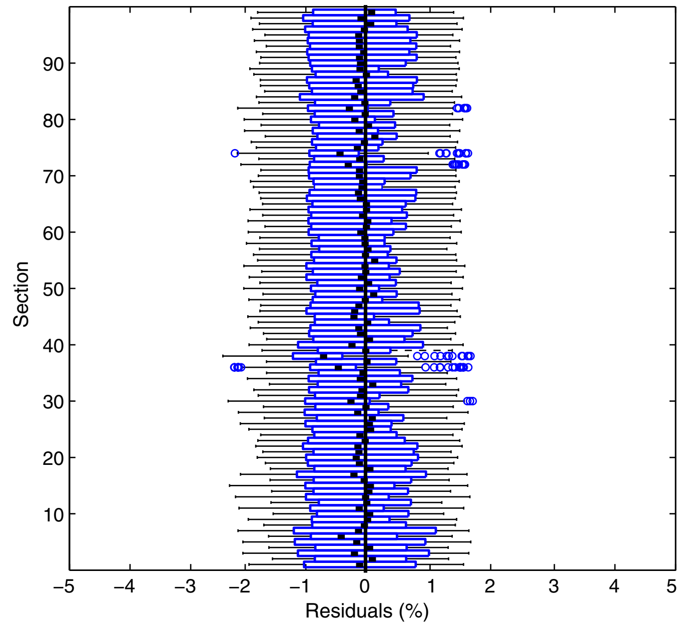

Fig. 10. Box plot of residuals from the nonlinear mixed-effect model

The graphical results for this model were similar to those obtained for model (4); residuals were approximately centered at zero and the normal distribution characterizes the residuals and the random effects. Finally, to conclude, according to both the AIC and BIC criteria, the best fitting model for MTQ data was model (5) with an AIC 1,548, which was marginally better than for model (4) (AIC of 1,553).

Simulated Data

Similar to MTQ data, the logistic mixed-effects model was also examined for simulated data. The intent of applying this model to simulated data was to study the effect of several factors, especially the maintenance effect, when inadequate information exists on the structural pavement in the real database (MTQ).

Fitting model (3) with only fixed-effect parameters resulted in an AIC of 86,071. The P-values for B (13.362) and C (3.27) indicated that both sets were statistically significant (both < 0.0001). Adding the random effects, model (4) with random effects for both fixed effects was also found to have an AIC of 30,897. A strong correlation (0.974) was found between both random effects (b_i and c_i). This correlation may allow for the possibility of eliminating one of these random-effect parameters. Therefore, three combinations are possible, including (b_i), (c_i), and (b_i, c_i). In all cases, the models containing mixed-effects (fixed and random parameters) performed better than the models with only fixed effects, and the model including all random effects was found to be the best with the lowest AIC and BIC values, as observed in Table 4.

Similar to MTQ data, the normal distribution of random effects parameters and residuals was examined. Fig. 8 shows a normal plot of estimated random effects and indicates that both random effects were normally distributed. Similarly, the normality of the residuals seems reasonable, as shown in Fig. 9. The boxplots of residuals by section is illustrated in Fig. 10, and indicates that the residuals were approximately centered at zero.

Analysis of Covariates

A logistic mixed-effects model that examines the influence of structural pavement factors on the cracking evolution was constructed using Akaike criterion and loglikelihood test for model selection. The primary question of interest for the simulated data was the effect of maintenance on the individual parameters (B_i, C_i). Depending on structural pavement conditions (for example, time of maintenance, thickness of the based layer), the influence of any other covariate could be studied.

Table 5 shows the model selection criteria for incorporation of covariates. Both statistical criteria of AIC and the likelihood ratio test (LRT) indicated the superiority of model (c) in which all covariates had a significant effect on the midpoint and scale parameters (B and C) whereas only one covariate (h_e) had no significant effect on C parameter for model (b) (see Tables 5 and 6).

Table 5. Sequentially Incorporating Covariates into Model (c) to Determine Important Covariate. Model Selection Using Backward Elimination Approach

Model	Covariates included	AIC	BIC	Test	LRT	P-value
a	None	30777	30820		—	
b	h_e, h, c_t in B, and C	30026	30113	a versus b	762.551	< 0.0001
c	h_e, h, c_t in B, and h, c_t in C	30024	30104	a versus c	726.458	< 0.0001
d	h_e, h, c_t in B, and h in C	30097	30170	a versus d	687.246	< 0.0001

Table 6. Results of Fitting Model (c) to Simulated Data

Parameter	Value	STD error	t-value	P-value
B (Intercept)	−35.901	0.722	−49.697	< 0.0001
$B.h$	131.706	1.639	80.318	< 0.0001
$B.h_e$	10.615	1.652	6.423	< 0.0001
$B.c_t$	−0.0247	0.003	−9.710	< 0.0001
C	−4.781	0.072	−66.131	< 0.0001
$C.h$	18.261	0.169	107.64	< 0.0001
$C.c_t$	0.002	0.005	10.615	< 0.0001

Table 7. Results of Fitting Model (d) to the Simulated Data

Parameter	Value	STD error	t-value	P-value
B (Intercept)	−33.611	0.926	−36.283	< 0.0001
$B.h$	131.706	2.252	58.461	< 0.0001
$B.h_e$	10.641	1.644	6.468	< 0.0001
$B.c_t$	−0.047	0.003	−42.270	< 0.0001
C	−4.526	0.099	−45.584	< 0.0001
$C.h$	18.289	0.247	73.784	< 0.0001

The positive sign of the coefficient associated with the thickness of the surface layer covariate indicates that this covariate increases the value of B and, consequently, has a delay effect on the occurrence of cracking. In other words, the pavement is able to better resist fatigue cracking after maintenance. In addition, the pavement structure also proved to be influenced by the thickness of the base layer (h): the positive sign of the coefficient associated with the h covariate indicated that the higher the value of h, the more mechanically robust the pavement and, therefore, cracking developed less rapidly. However, the parameter coefficient associated with the time of maintenance (c_t) dictates the shape of the curve (b). The longer that maintenance is delayed, the faster cracking develops. Thus, maintenance exerts a substantial effect on evolution speed. The positive sign of the coefficient associated with the covariate c_t of C does not have physical significance, which is shown as a shortcoming of the model.

As a result, the covariate c_t was not accounted for in the C parameter of the model that yielded the estimates shown in Table 7. These estimates, along with the diagnostic plot, have the same interpretation as that of the model including c_t.

To identify the best of the two models [model (c) and model (d)], a comparison based on the prediction ability was carried out and achieved by partitioning the data set into two subsets. The first

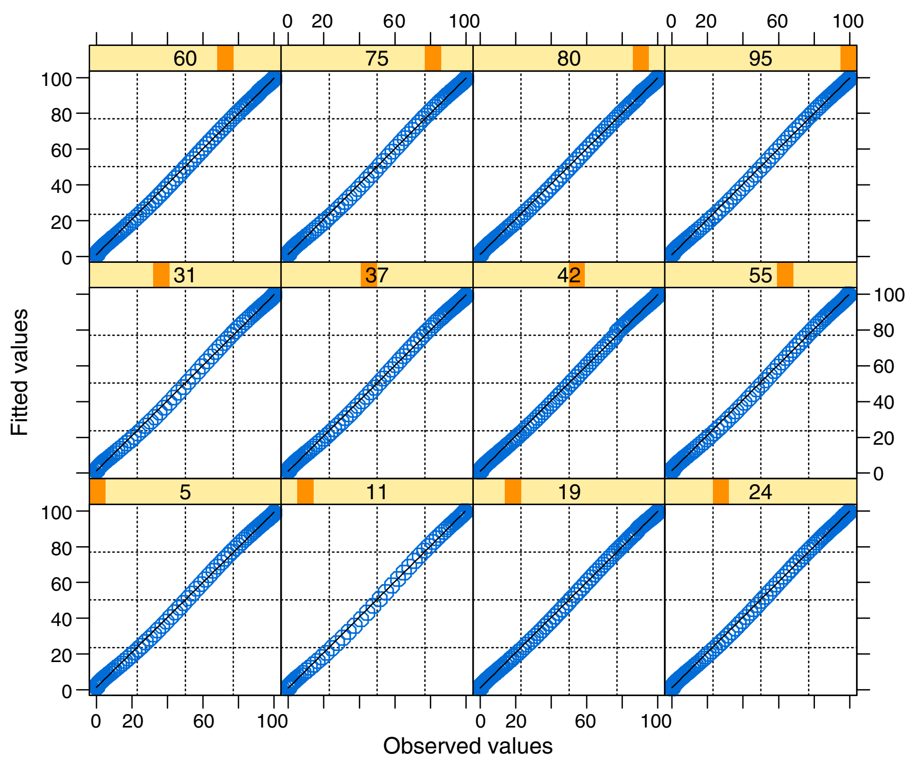

Fig. 11. Fitted values plotted against observed values for each of 12 from 100 pavement sections in the simulated data

subset, containing data below 20% of cracking, was used for fitting. The other subset served to determine the relative mean error (RME). The obtained results showed that the two models have almost the same prediction ability ($\mathrm{RME}_c = 10.8\%$ and $\mathrm{RME}_d = 10.7\%$) and justified, the choice of model (*d*), as it not only has better prediction ability, but also a plausible physical interpretation.

Finally, the inclusion of the pavement structure factors in the logistic mixed-effects model resulted in a reduction in the estimated standard deviation for the *B* random effects from 3.620 to 0.609 and in the estimated standard deviation for the *C* random effects from 0.499 to 0.064, indicating that a substantial part of the plot-to-plot variation in these two parameters was explained by differences in thickness of the base and surface layer. To illustrate the goodness of fit, Fig. 11 shows 12 plots from a total of 100 plots. The plot-specific estimates were close to the observed values, indicating that the logistic mixed-effects model (*d*) adequately represented the simulated data.

Conclusion

This paper developed a mixed-effects logistic model for describing the evolution law of pavement deterioration and for identifying the effects of several factors on pavement behavior. Using real data (MTQ), the environmental factor was found to have a significant influence on cracking progression.

In simulated data, the maintenance effect had a greater influence on cracking progression. Restoring the structural capacity of flexible pavements through timely maintenance intervention may assist in delaying the rate of deterioration.

Traditional regression models assume that observations are independent and identically distributed; in case of longitudinal road data studies, this hypothesis is no longer valid. Thus, having a recourse to mixed models is necessary, which assume two sources of variation, within and between sections. This decomposition of variation leads to valid statistical estimations of the model parameters.

This study showed the effectiveness of the logistic mixed-effects model as a new approach to explain the pavement cracking data. Furthermore, this approach made optimum use of the data by taking into account unit-to-unit variability; consequently, the approach was found to be more powerful than traditional regression approaches to establish the evolution curves of each pavement section and to identify the most important factors involved in the cracking process.

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